**Papers written by**

**Australian Maths**

**Software**

**SEMESTER ONE**

**MATHEMATICS SPECIALIST REVISION 3**

**UNIT 3**

**2016**

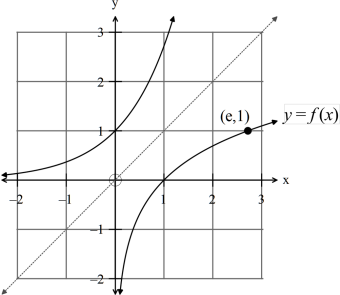
**SOLUTIONS**

Mathematics Specialist Unit 3, 2016, Semester One Solutions

**Section One**

1. (6 marks)

(a) (i)

✔ (1)

(ii) *f* ( *x*) = *ln*( *x*) ✔

( ) ✔ (2) 1 *x f x e* − =

(iii) ✔ (2) 1 − ∈ > ∈ *f : x R y* 0*, y R* ✔

(iv) If then ( ) ✔ (1) 2 *f e* = 2 ( )

− =

1 2

*f* 2 *e*

2

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2. (10 marks)

⎛ ⎞

2 0 0 2 ⎜ ⎟

0 5 0 10

(a)

− ⎝ ⎠

0 0 1 3

*z*

= −

3

5 10 2

*y y*

= ⇒ = 2 2 1

*x x*

= ⇒ = ( )

Point of intersection is 1 2 3

*, ,*

−

(1)

(b)

✔

⎛ − ⎞

1 2 1 3 ⎜ ⎟ 0 2 0 1

0 0 0 2

⎝ ⎠

0 = 2 No solution ✔

The lines of intersection of any two of the planes are parallel. ✔ (2)

⎛ − − ⎞

1 2 5 2

⎜ ⎟

0 1 3 6

(c)

0 0 0 0

⎝ ⎠

0 = 0 An infinite number of solutions. ✔

Two of the planes could be identical and intersect the third plane or the

three planes intersect in one line. ✔ (2) (d)

1 1 1 0

⎡ ⎤

⎢ ⎥ − −

2 1 2 6

⎣ − − ⎦

2 2 1 5

1 1 1 0

⎡ ⎤

⎢ ⎥ −

0 3 0 6 2

*R R*

✔

1 2

⎣ − − − ⎦ −

0 4 1 5 2

*R R*

✔

3 1

1 1 1 0

⎡ ⎤

⎢ ⎥ − − −

0 4 1 5

Swap rows 2 and 3

0 3 0 6

⎣ ⎦

✔

3 6 2

*y y*

= → = ( )

− − = − → = − 4 2 5 3

*z z*

*x x*

+ − = → = 2 3 0 1

✔

( )

✔

1 2 3

*, ,*

−

The point of intersection is

(5)

3

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3. (8 marks)

(a) (i) ( ~~)~~ ✔✔ (2) ( )

*x*

−

1

*f ~~x~~*

= ~~−~~

( )

*x x*

−

2

(ii) ( ~~)~~ ✔✔✔ (3) ( )

( )

2 1 2 1

*x x*

− −

*f ~~x~~*

= ~~=~~

( )( )

( )

2 2 2 4 1

*x x x x*

− −

(iii) ( ~~)~~ ✔✔✔ (3) ( )

*x x*

−

2

*f ~~x~~*

=−

( )

*x*

1

4. (13 marks)

( ) ( ( ))( ( ))( )

a *z i z i z*

− + − − − =

1 2 1 2 4 0

✔

(( ) )(( ) )( )

*z i z i z*

− − − + − =

1 2 1 2 4 0

( )( )

2 2

*z z i z*

− + − − =

2 1 4 4 0

( )( )

2

*z z z*

− + − =

2 5 4 0

3 2

*z z z*

− + − =

6 13 20 0

✔

∴ *a* = −6*, b* =13*, c* = −20 ✔ (3)

(b) 3 2

*z* + 2*z* + 2*z* +1 = 0

Using synthetic division with You can use long division but slower z = −1

∴= − +

*z z*

3 2

*z z z*

1 1soisafactor

+ + + =

2 2 1 0

2

*z z*

−

++

1

1 1 2 2 1

)

3 2

*z z z*

+++

2 21

*z*

+

✔ method

1

↓− − −

1 1 1

( )

3 2

− +

*z z*

1 1 1 0

2

✔

∴ = − + + = *z z z*

1 1 0

2

*z z*

+

2

OR

( )

− ± −

1 1 4

2

*z*

= 2

− +*z z z*

+

1

2

− ± − − ± 1 3 1 3

*i*

*z*

( )−+

*z* 1

= ~~=~~

✔

2 2 0

− ±

1 3

*i*

✔

*z*

=

2

*z z z* = − ++=1 10

2

or (4)

4

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(c) (i) 3

*z* = −8

( )

3

*z cis*

= π

8

( )

3

*z cis n n R* = π + π ∈ 8 2

1

( ( ))

*z cis n*

= π + π

2 2

3

⎛ π + π ⎞ = ⎜ ⎟

2

*n*

*z ~~cis~~*

23

✔

⎝ ⎠

⎛ π ⎞ = ~~=~~ = + ⎜ ⎟ *n ~~z cis~~ i*

0 ~~2~~ 1 3

***,***

3

⎝ ⎠

⎛ π ⎞ = ~~=~~ = π = − + ⎜ ⎟

3

( )

*n ~~z cis~~ cis i* 1 ~~2~~ 2 1 3

***,***

3

⎝ ⎠

⎛ π ⎞ = ~~− = −~~ = − ⎜ ⎟ *n ~~z cis~~ i*

1 ~~2~~ 1 3

***,***

✔✔ -1/error

3 ⎝ ⎠

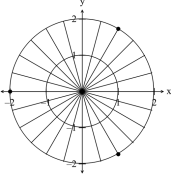
2

π

(3)

(ii) The roots are apart around the origin. ✔ (1) 3

(iii)

✔✔ -1 per error 

(2)

5

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5. (13 marks)

4 4 4 2 − 2*i* = 2 1− *i*

(a) (1) ( ) ( )

4

⎛ ⎛ π ⎞⎞ = ⎜ ⎜− ⎟⎟

16 ~~2~~

*cis*

4

⎝ ⎝ ⎠⎠

( )

= × × −π

16 4

*cis*

( )

= − +

64 1 0

*i*

= −

64

✔

(b) 3 2 1

− +

*i i*

− ~~+~~

3 1

+ −

*i i i*

3 3 2 1 1 − − + +

*i i i i i*

= ~~× − × + ×~~

✔

3 3 1 1

+ − − +

*i i i i i i*

8 6 1 3

− +

*i i*

= ~~−~~ −

*i*

✔

10 2

3 31

−

*i*

✔

=

10

(3)

( ) ( )( )

c *x yi i i*

+ = + −

2 3 3 4

2

= + − −

6 9 8 12

*i i i*

= +

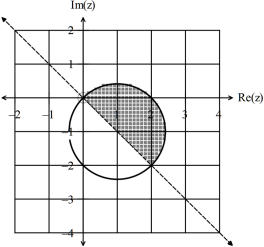
18

*i*

*x , y*

= =

18 1

✔ ✔ (2) (d) 

✔

-1 per error

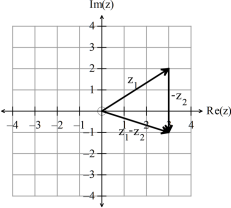
✔

(2)

6

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(e)

✔✔

(2)

4

⎛ ⎛ π ⎞⎞

*cis*

⎜ ⎜ ⎟⎟

4

( ~~)~~ ( ~~)~~

*zz ~~cis~~*

424

⎝ ⎝ ~~⎠~~⎠ ⎛ π ⎞ × ~~= ×~~ ⎜− ⎟

1

f

3 3 3

( )

⎛ ⎛ π ⎞⎞ ⎝ ⎠

*zcis*

2

⎜ ⎜ ⎟⎟

3

⎝ ⎝ ⎠⎠

⎛ π ⎞

4

*cis*

⎜ ⎟

⎝ ⎠ ⎛ π ⎞ = ~~× ×~~ ⎜− ⎟

4

23 4

*cis*

✔

⎛ π ⎞ ⎝ ⎠

*cis*

⎜ ⎟

3

⎝ ⎠

⎛ π ⎞ = ~~×~~ ⎜π ~~−~~ − π⎟

24

*cis*

⎝ ⎠

⎛ π ⎞ = ~~×~~ ⎜− ⎟

24

*cis*

✔

⎝ ⎠

1

*i*

⎛ ⎞ = ~~× −~~ ⎜ ⎟

22 2

⎝ ⎠

= −

1

*i*

✔

4

( )

*zz i*

1

× = −

1

3 3

( )

*z*

2

(3)

**END OF SECTION ONE**

7

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**Section Two**

6. (19 marks)

(a) (i) ***r***1 (*t*) = (*sin*(*t*))***i*** +(*cos*(*t*)) ***j*** and ***r***2 (*t*) = (*sin*(*t*))***i*** −(*cos*(*t*))***j***. ( ) ( ) ( ) ( )*x sin t y cos t x sin t y cos t* = = = =−

✔

( ) ( )

2 2

*sin t cos t*

+ =

1

✔

✔

2

( ) (())

2 2 2 2 2

⇒ + = + = +−*x y x y sin t cos t* 1

( ) ()

2 2

✔

= +

*sin t cos t*

2 2

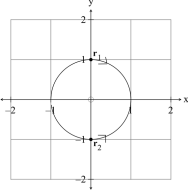
∴+ =

*x y*

1

(5)

(ii)

✔ 

✔✔

✔

(2)

0 0

⎛ ⎞ ⎛ ⎞ = ⎜ ⎟ = ⎜ ⎟ ( ) ( ) ( ) **r r**

0 0

iii

1 2

⎝ ⎠ ⎝− ⎠

1 1 See diagram (2)

⎛ ⎞ ⎛ ⎞ = ⎜ ⎟ = ⎜ ⎟

+ +

0 0

( ) ( )

+ +

**r r**

0 0

1 2

− −

⎝ ⎠ ⎝− ⎠ 1 1

⎛ ⎞ ⎛ ⎞

3 3

⎜ ~~⎟ ⎜~~ ⎟

⎛ π ⎞ ⎛ π ⎞ = ⎜ ⎟ − = ⎜ ⎟

2 2

(iv) 1 2 ✔

**r ~~r~~**

⎜ ~~⎟ ⎜~~ ⎟

3 1 3 1

⎝ ⎠ ⎜ ⎟ ⎝ ⎠ ⎜ ⎟ ⎜ ⎟ ~~⎜~~ − ⎟ ⎝ ⎠ ⎝ ⎠

2 2

Distance apart is 1 unit. ✔

(2)

8

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(b) (i) ***r***(*t*) = (6 *sin*(*t*) + 2*cos*(*t*))***i*** + (6 *sin*(*t*) − 2*cos*(*t*))***j***. ( ) ( ( ) ( )) ( ( ) ( )) ***v i j***

*t cos t sin t cos t sin t* = − + + 6 2 6 2

✔✔

( ) ( ( ) ( )) ( ( ) ( ))***a i j*** *t sin t cos t sin t cos t* = − − + − + 6 2 6 2

✔

( ) (( ( ) ( )) ( ( ) ( )))***a i j*** *t sin t cos t sin t cos t* = − + + − 6 2 6 2

✔

( ) ( )

***a r***

*t t*

= −

(4)

(ii)

( ) ( )

( ) ( )

⎛ + ⎞ ⎛ − ⎞ ∙ = ⎜ ⎟ ∙ ⎜ ⎟ − + ⎝ ⎠ ⎝ ⎠

6 2 6 2

*sin t cos t cos t sin t*

✔

( ) ( )

***r v***

*t t*

( ) ( )

( ) ( )

6 2 6 2 *sin t cos t cos t sin t* ( ( ) ( )) ( ( ) ( )) = + × − 6 2 6 2 *sin t cos t cos t sin t* ( ( ) ( )) ( ( ) ( )) + − × + 6 2 6 2 *sin t cos t cos t sin t* ✔

( ) ( ) ( )

2 − 12*sin t* ( ) ( )

( )

2

= +

36 12

*sin t cos t cos t*

−

4

*sin t cos t*

+ − ( ) ( ) ( ) ( )

2 + 12*sin t* ()()

2

36 12

*sin t cos t cos t*

−

( ) ( ) ( ) ( ) ***r*** ∙ ***v*** = *t t sin t cos t* 64

✔

4 *sint cost*

(3)

(iii) If 0 then =0 ***r***(*t*)∙ ***v*** (*t*) = 64*sin*(*t*) *cos*(*t*)

32 (2 ) 0

*sin t*

=

✔

2 0 2

*t*

= π π

***, ,***

π

*t*

= π

02

***, ,***

✔

(3)

7. (7 marks)

(a) (*t*) = ( *cos*(5*t*) + *sin*(5*t*) ) *dt* ∫

***r i j***

( ) ( )

*sin t cos t*

5 5 1 ⎛ π ⎞ = ~~− +~~ ⎜ ~~⎟~~ = ~~−~~⎝ ⎠

**i j ~~c r~~ j**

✔

But

5 5 5 5 **j ~~i~~** ( )

( )

*sin*

π

*cos*

π

1

∴− ~~=~~

− +

**j c**

✔

5 5

5

1 1 2

− ~~= + ⇒= −~~ **j j ~~c c~~ j**

✔

5 5 5

( ) ( )

*sin t cos t*

5 2 5

+

( ~~)~~

**r ~~i~~ j**

*t*

= ~~−~~

5 5

(3)

9

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∫

( ) ( ) ( ( ) ( ) ) b *t cos t sin t dt* ***r i j***

= +

5 5

( ) ( ) ( )

***v i j***

*t cos t sin t*

= +

5 5

✔

( ) ( ) ( )

***a i j***

*t sin t cos t*

= − +

5 5 5 5

✔

(2)

( ) ( )

*sin t cos t*

5 5 2 3

( ~~)~~ ( ~~)~~

( ~~)~~

c **r ~~i j j r~~ j** *t*

= ~~− − ⇒= −~~ 0

✔

5 5 5 5 ( ) ( ) ( ) ( )

**v i j v i**

*t cos t sin t*

= + ⇒= 5 5 0

✔

(2)

8. (3 marks)

⎛ ⎞ ⎛− ⎞

0 3

⎜ ⎟ ⎜ ⎟ = − = −

**AB AC**

1 1

*,*

✔

⎜ ⎟ ⎜ ⎟

0 0

⎝ ⎠ ⎝ ⎠

**AB AC i j k**

× = + +

0 0 3

0

⎛ ⎞

⎜ ⎟ =

✔

0

⎝− ⎠

3

0

0

⎛ ⎞

⎛ ⎞

⎜ ⎟

✔

⎜ ⎟

Therefore the unit vector required is NB is also OK

0

0

⎝− ⎠

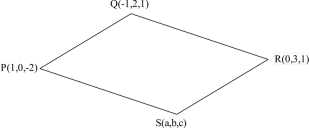
1

1

⎝ ⎠

(3)

9. (6 marks)

(a) *P*(1***,*** 0***,*** − 2) ***,****Q*(−1***,*** 2***,*** 1) and *R*(0***,*** 3***,*** 1) and *S* (*a****,*** *b****,*** *c*) 

⎛− ⎞ ⎛ − ⎞

2

*a*

⎜ ⎟ ⎜ ⎟ = = −

***PQ , SR***

2 3

*b*

⎜ ⎟ ⎜ ⎟

✔

− ⎝ ⎠ ⎝ ⎠

3 1

*c*

***PQ SR***

=

∴ = = − ⇒= = − ⇒= − *a b b c c*

2 2 3 1 3 1 2

***, ,***

( )

∴ −

*S*

2 1 2

***, ,***

✔✔

(3)

10

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(b) (i)

⎛ ⎞ ⎛− ⎞

2 1

⎜ ⎟ ⎜ ⎟ × = × ***a b***

0 3

⎜ ⎟ ⎜ ⎟

3 4

⎝ ⎠ ⎝ ⎠

✔

***a b i j k*** × = − − + 9 11 6

(1)

( ~~)~~ ( ) 12 ii *Area*Δ = *a*×*b*× *sin C*

1

*Area*

= ×

***a b***

as where is the angle ***a***×***b*** = ***a b*** *sin*(θ) θ✔

10. (7 marks)

Δ

2

between ***a*** and ***b***.

⎛ − ⎞

9

111

⎜ ⎟ = −

26

⎝ ⎠

181 121 36

= + +

2

1238

=

2

✔

2

*Area units*

=

7 71

***.***

Δ

1

⎛ ⎞

⎜ ⎟

(a) (i) Use to find vector between P(1,0,1) and .✔

*A*(1***,***2***,***3)

0

1

⎝ ⎠

0

⎛ ⎞

⎜ ⎟ =

✔

***AP***

2

2

⎝ ⎠

1 1 0

⎛ ⎞ ⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = +

Equation of plane is ( ) ✔

***r***

*t +t s*

0 4 2

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

1 3 2

⎝ ⎠ ⎝ ⎠ ⎝ ⎠

(3)

1 1 0

⎛ ⎞ ⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = +

***r*** *M*(6***,***20***,***16) (ii) Test ( )

*t +t s*

0 4 2

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

1 3 2

⎝ ⎠ ⎝ ⎠ ⎝ ⎠

*x t*, *y t s*, *z t s*

= + = + = + +

1 4 2 1 3 2

*x t y s z s*

= ⇒ = = + = + 6 5 20 2 16 2

If so

*y s* ∴*z*

= ⇒ = = 20 0 16

If

Yes, the point *M* (6***,***20***,***16) belongs to the line. ✔ (1)

11

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(b) (i) John:

⎛ ⎞ ⎛− ⎞ ⎛ ⎞

3 1 0

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ + =

2 1 5

*t*

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

0 1 3

⎝ ⎠ ⎝ ⎠ ⎝ ⎠

3 0 3

− = ⇒ =

*t t*

2 5 3

+ = =

*t Yes t*

***,***

0 3 3

+ = =

*t Yes t*

***,***

John takes 3 seconds to reach the parcel. ✔ (1)

(ii) James:

⎛− ⎞ ⎛ ⎞ ⎛ ⎞

2 1 0

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ + =

1 2 5

*t*

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

0 1 5 3

***.***

⎝ ⎠ ⎝ ⎠ ⎝ ⎠

− + = ⇒ = 2 0 2

*t t*

1 2 5 2

+ = =

*t Yes t*

***,***

0 1 5 3 2

+ = =

***. ,***

*t Yes t*

James takes 2 seconds to reach the parcel so he gets to

the parcel first. ✔ (1)

(iii) John: James

⎛− ⎞

1

1

⎛ ⎞

⎜ ~~⎟~~ =

⎜ ~~⎟~~ =

1 3

2 7 25

***.***

1

1 5

***.***

⎝ ⎠

⎝ ⎠

James moves with the greater speed. ✔ (1)

11. (6 marks)

2 1

*m*

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ ∙ − = ⇒ − + = ⇒= − *m m m m*

1 0 2 6 0 6 (a)

✔

⎜ ⎟ ⎜ ⎟

2 3

⎝ ⎠ ⎝ ⎠

✔ (2) 2 2 2

(b) (i) ( ) ( ) ( )

*x* −1 + *y* + 3 + *z* − 2 = 25

✔ ( )

Substitute *P*

3 1 1

***, ,***

2 2 2

( ) ( ) ( )

3 1 1 3 1 2 4 16 1 21 25

− + + + − = + + = < The point is INSIDE the circle. ✔ (2)

⎛ − ⎞

*x*

1

⎜ ⎟ + = − + + + − = (ii) ( ) ( ) ( ) ✔✔ (2) *y or x y z*

3 5 1 3 2 5

***i j k***

⎜ ⎟

− ⎝ ⎠

*z*

2

12

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12. (6 marks)

(a) ***v*** (0) = ***i r***(0) = ***j***

✔

( )

***a . j***

0 9 8

= −

***v( ) . j c***

*t t*

= − +

9 8

***1***

*At t*

= =

0

***i c***

***1***

✔

∴ = −

***v( ) i . j***

*t t*

9 8

2

***r( ) i . j c***

*t t t*

= − +

4 9

2

*At t*

= =

0

***j c***

2

( )

2

∴ = + −

***r( ) i . j***

*t t t*

1 4 9

✔

2

*At h x h t*

= = = −

0 1 4 9

,

***? .***

✔

1 0

2

*t t*

= >

4 9

***.***

*t*

=

0 4517539515

***.***

*x t so x m* = =

0 45

***.***

✔

The ball hits the floor 0.45 m from the table. (5) (b) The ball took 0.45 seconds to hit the floor. ✔ (1)

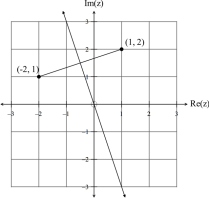
13. (12 marks)

⎧ π π ⎫ (a) -1/error (1 ) 2 ( ~~)~~ 4 2

⎨ − ~~+ ≤ ∩ < ≤~~ ⎬

*z*: *z ~~i~~ arg ~~z~~*

⎩ ⎭ ✔ ✔ ✔ correct inequalities (3)

(b) (1,2) (-2,1) midpoint is (-0.5,1.5) *z* −1− 2*i* = *z* + 2 − *i *

✔✔✔ (3)

13

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(c) (i) (1+ 3*i*)(1+ *i*) = (1− 3) + (1+ 3)*i* ✔ (1) (ii) If show that and *z* = (1+ 3*i*)(1+ *i*) *z* = 2 ~~2~~ ( ~~)~~ 34ππ*arg ~~z~~* =+*.* ( )( ) ( ) ( )

1 3 1 1 3 1 3

+ *i* + *i* = − + + *i*

✔

2 2

( ) ( )

= − + +

1 3 1 3

= − + 3+1+ 2 3 3

1 2 3

+

✔

=

8

=

2 2

( ) (( )( )) *arg z arg i i*

= + +

1 3 1

( ) ( )

= + + +

*a rg i arg i*

1 3 1

π π π

7

( ~~)~~

*arg ~~z~~*

= ~~+ =~~

✔

3 4 12

(3)

(iii) Show that 7 1 3

⎛ π ⎞ +

⎝ ⎠ *sin .*

⎜ ~~⎟~~ =

12 2 2

7 ( )

*Im z*

⎛ π ⎞

✔

*sinr*

⎜ ~~⎟~~ =

12

⎝ ⎠

from (c) (i)

1 3

+

=

✔

from (c) (ii)

2 2

(2)

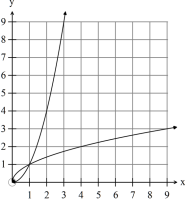
14

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14. (17 marks)

(a) (i) The inverse exists because the function is a one to one function so for every x value, there is a unique y value. ✔✔ (2)

(ii)

✔ 

( )

− =

1

*y g x*

✔

*y* =*g* ( *x*) =*x*

(2)

(iii) ( ) ( ) ✔✔ (2) 1 2 1 − − = = ≥ = ≥ *y g x x x* 0 *y g x* 0

(iv) ( ) ✔ (1) 1

− =

*g* 4 16

(b) Show that *f* ( *g* ( *x*)) = *g* ( *f* ( *x*))*.*

✔

( ( )) ( ) ( )

*f g x f x x x* = − = − − = −

2 2 2 1 3 2

( ( )) ( ) ( ) ( ( )) *g f x g x x x f g x* = − = − − = − = 2 1 2 2 1 3 2

✔

(2)

(c) (i) ( ( )) ( ) ( ) ✔ ✔ 2 2 2 *y* = *p q x* = *p x* − 3 = 1− *x* − 3 = 4 − *x* which is defined for −2 ≤ *x* ≤ 2*.* ✔ (3)

2

( ) ( ( )) ( ) ( )

✔

*y q p x q x x x* = = − = − − = − −

ii

1 1 3 1 3

( ( ))

*y q p x x x* = = − − ≤ 2 1

for

✔

*q*( *p*(2)) = *q*( 1− 2 ) = *q* −1 which is not defined. ✔ (3) (iii) The range of is *y* = *q* ( *p*( *x*)) *y* ≥ −3*.* ✔✔ (2)

15

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15. (8 marks)

(a) (i) x = 1 or x = -1 ✔ (1) (ii) 21

*x* < *. x*

**

✔ ✔

(2)

⎧ +

1

*x*

for x

≤

0

( ) ( )

✔

*f x x*

= − = ⎨⎩ −

11

b

*x*

for x>

0

1

for x

<

⎧ −

1 2 2

*x*

( )

*g x*

*x*

= − = ⎨⎩ − +

1 2

✔

1 2 1

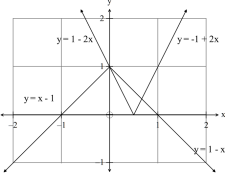
*x*

for

x

≥

2

✔ 

112

−=−+

*xx*

23

*x*

==

*x*

2

3

203*x* =*or ~~x~~* =

✔

✔

(5)

16. (4 marks)

✔ ✔

(4) ( )( ) *yx x* − +

*x x*

1 1

=−

( )

2

✔ ✔

16

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17. (5 marks)

Prove that . ( ) ( ) ( ) ( ) ( ) 3 3 *sin* 4θ = 4*cos* θ *sin* θ − 4*cos* θ *sin* θ

✔

( ) ( ( ))

*sin Im cis*

4 4

θ = θ

4

✔

( ( ))

= θ

*Im cis*

4

( ( ) ( ))

= θ + θ

*Im cos isin*

1 4 6 4 1

4 3 2 2

✔

( ( ( )) ( ( )) ( ( )) ( ( )) ( ( ))= θ + θ θ + θ θ*Im cos cos isin cos isin* 4 6

✔

3 4

( ( ))( ( )) ( ( )) )

+ θ θ + θ

4

*cos isin isin*

( ( ) ( ) ( ) ( ) ( ) ( ) ()())

4 3 2 2 2 3 3 4 4= θ + θ θ + θ θ + θθ+θ

*Im cos i cos sin i cos sin i cos sini sin*4 6 4 ( ( ) ( ) ( ) ( ) ( ) ( ) ()())

4 3 2 2 3 4 = θ + θ θ − θ θ − θθ+θ

*Im cos i cos sin cos sin i cos sinsin*4 6 4

✔

( ) ( ) ( ) ( ) ( )

3 3 ∴ *sin* 4θ = 4*cos* θ *sin* θ − 4*cos* θ *sin* θ (5)

**END OF SECTION TWO**

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